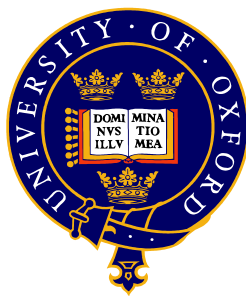


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AN INVESTIGATION OF CERTAIN THERMODYNAMIC LOSSES IN MINIATURE CRYOCOOLERS

FINAL REPORT

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2 EXECUTIVE SUMMARY

A detailed investigation of these losses was made using different gases over a range of frequencies, and the losses were estimated from the 'pdV' work done by the piston on the gas in the cylinder.

Having allowed for the pumping loss due to the clearance seal, the work done on the gas was evaluated, and this data was compared with expressions derived by Kornhauser and others using sliding seal compressors.

An important conclusion is that these expressions, derived for conventional sliding seal compressors, are valid for clearance seal machines, once the seal pumping loss has been allowed for. Other conclusions that can be drawn from these results are:

- For Peclet numbers in the range 10 to 1000, Kornhauser's modified version of Lee's equation gives good results and can be used in applications involving typical clearance seals.
- There appears to be a significant deviation from Lee's expression for all but the lightest gases (i.e. Helium and Hydrogen) where the Peclet number is above 1000. It would be of interest and value to investigate this further to discover the cause and whether there is a change in flow regime that can be defined by a type of Reynolds number.

It was intended to use a multiple thermocouple technique to investigate, but practical difficulties were encountered when this technique was attempted, and this avenue of investigation was abandoned, though the technique does remain an attractive option to try in the future.

The original intention of the project was, having established a 'baseline' for the compression loss in a plain volume, to investigate the extra losses when other geometries, such as heat exchangers, were added to the test system. This idea of adding components to a simple gas spring in order to measure their compression losses was found to be flawed. However a more sophisticated approach, that involves varying the extent of a heat exchanger and relating this to changes in the measured losses, is described and does appear to be viable. This new approach requires a heat exchanger component that was not designed into the original test rig. It also requires the regenerator to be repacked several times for a set of results. It was not possible to obtain any measurements in this program but it is still considered worth pursuing in any future program.

3 BACKGROUND

This report describes work which is a continuation of that documented in report Los-Fr1¹. The aim has been to investigate thermodynamic losses that appear to occur in miniature Stirling cycle coolers but which are not adequately explained by our existing models. These losses are large enough that a small reduction in their value would produce a significant improvement in cryocooler efficiency.

It is believed that these losses are mainly associated with the heat transfer and flow processes that occur when the cooler components are subject to a cyclic pressure variation and are not a direct result of the refrigeration process. This has suggested that there would be equivalent losses in systems that have similar pressure and flow processes but which do not have the usual mechanisms for generating refrigeration, such as a second piston or displacer. Such systems would be similar to gas springs but with components and geometries that derive from cooler designs rather than those typically selected for gas spring efficiency.

If there is no refrigeration cycle operating then all power dissipated can be attributed to the loss mechanisms including those that are of interest. The measurement of the power dissipated for differing heat transfer and flow regimes should therefore allow the correlation of loss values with variations of particular parameters in a way that would not be possible in an actual cooler.

At the beginning of this project it was believed that measurements of both the electrical power input (after losses are subtracted) and the PV power measured in the compression space could give useful values for the losses. The measurement of the PV power is more direct but requires the installation of a pressure transducer and needs care to avoid signal processing errors. The use of the electrical power input would be more generally applicable but can only be accurate if a thorough breakdown and analysis of other losses is possible.

In the first report the design and construction of a test rig was described. Work was also described in which a detailed analysis of all the losses was made. This allowed a comparison to be made between the measured PV power and the equivalent power deduced from the electrical power input. The conclusions drawn were:

- There were contributions from losses other than coil resistive heating that introduced significant uncertainty in the loss values calculated from electrical input power.
- The careful measurement of phase lags in the signal processing allowed pressure and displacement values to be corrected.
- The direct measurement of PV power through the use of a pressure and displacer transducers was the more accurate and reliable method.

In the work described below, values for compression losses are obtained by measuring the integrated PdV power and then subtracting calculated values for the seal losses.

4 INTRODUCTION

The experimental apparatus used for the initial work described in “Los-Fr1” was a modified linear compressor connected to a cylindrical compression volume as shown in figures 1 and 2. This design represents the simplest geometry for a gas spring. Although it was not the original intention to investigate this simple geometry in detail, it became apparent that useful measurements could be made.



Figure 1
Test Rig Used For Gas Spring Loss
Measurements

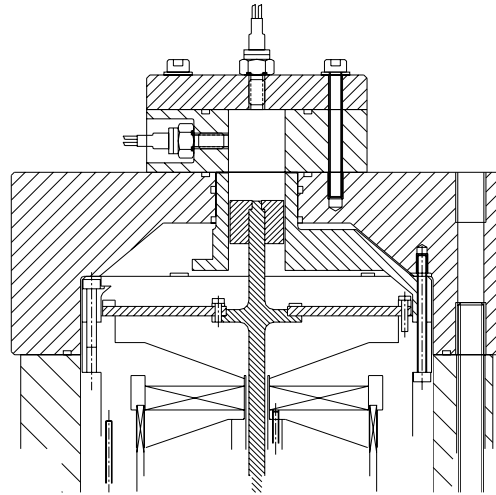


Figure 2
Section through Test Rig Showing
Gas Spring Geometry

Our present understanding of gas spring losses is largely based on the work of Kornhauser, Smith and others² who carried out a series of thorough investigations on conventional reciprocating compressors, where the use of normal sliding seals would minimise seal losses. The widespread use of linear clearance seals in linear compressor has raised the question of how applicable the Kornhauser correlations are to compressors with clearance seals, which have significant flow through the seals and consequent seal losses.

This project offers a good opportunity to extend Kornhauser’s work to include clearance seal type compressors. The first part of this report describes loss measurements for a simple gas spring geometry and a comparison with Kornhauser’s results.

The measurement of instantaneous temperatures offers the possibility of more a detailed understanding of heat transfer processes, especially with regard to:

- Boundary layer measurements
- Enthalpy flow between components

Achieving rapid response times with conventional thermometry is problematic because rapid response is achieved by using very small probes. Small probes are difficult to handle and

those of a manageable size do not really have a fast enough response. A novel technique that allows the frequency response of a sensor to be significantly increased has been successfully used in engine combustion research³. This approach looks to be one that could be usefully applied to the study of compression losses. The second part of this report gives a short description of some work done on this area.

The longer term goal of this project is to use this approach to gain insight into the losses associated with typical cooler components. The third part of this report describes initial work done looking at heat transfer and pressure drop losses in typical regenerators

5 COMPRESSOR LOSSES IN A SIMPLE GAS SPRING GEOMETRY

5.1 The Work of Kornhauser and Others

An important reference for this subject is the 1987 paper of Kornhauser and Smith. This paper presents experimental data from a test rig which allowed loss measurements to be made over a wide range of operating conditions. This data was compared with the predictions of a number of theoretical treatments and the conclusion drawn was that the treatments by Lee⁴ and by Cooke-Yarborough⁵ were the most accurate. The equation finally adopted by Kornhauser was Lee's.

In Kornhauser & Smith's later paper of 1993⁶ the work was extended to wider range of conditions including finned geometries. Lee's expression was found to correlate much of the data fairly well but an empirical modification is proposed that appears give a significant improvement. The measurements that did not correlate well were those for high volume ratios and high molecular weight gases. It is suggested that the Lee's assumptions were basically less applicable for these conditions. It was also suggested that the appendix gap (the small annular gap around the piston) might have a significant influence. In clearance seal compressors the appendix gap has a significantly greater flow through it and it might be expected that it would influence the overall loss values.

Unfortunately there are one or two discrepancies in the papers of Kornhauser and Cook Yarborough that can cause confusion. These are described in Appendix 1.

5.1.1 Gas Spring Losses – Kornhauser 87

The equation adopted by Kornhauser in the 1987 paper is basically that presented by Lee:

$$\Delta \dot{W} = -\frac{\omega}{4} \cdot p_1 V_0 \frac{(\gamma - 1)}{\gamma} \cdot \frac{p_1}{p_0} \frac{1}{y_L} \cdot \left(\frac{\cosh(y_L) \cdot \sinh(y_L) - \sin(y_L) \cdot \cos(y_L)}{\cosh^2(y_L) \cdot \cos^2(y_L) + \sinh^2(y_L) \sin^2(y_L)} \right)$$

Kornhauser chooses to define the gas spring loss in terms of a non-dimensional loss constant L_{nd} defined as:

$$L_{nd} = \frac{Loss}{W_{adiab}} = \frac{\Delta \dot{W}}{W_{adiab}}$$

Also Kornhauser uses a definition for the parameter y which is a factor of two different to that used by Lee (see Appendix 1) so that with some algebraic simplification the expression arrived at is:

$$L_{nd} = \frac{p_0 \cdot V_0}{W_{adiab}} \cdot \frac{\pi}{2} \left(\frac{p_a}{p_0} \right)^2 \cdot \frac{(\gamma - 1)}{\gamma} \cdot \frac{1}{y} \left(\frac{\cosh(y) \cdot \sinh(y) - \sin(y) \cdot \cos(y)}{\cosh^2(y) - \sin^2(y)} \right)$$

Where
$$y = D_h \cdot \left(\frac{\omega}{32 \cdot \alpha_0} \right)^{1/2} = \left(\frac{\pi \cdot Pe_h \cdot D_h}{32 \cdot L_s} \right)^{1/2}$$

p_0 is Mean cycle pressure

V_0 is Mean cycle volume

W_{adiab} is Work of adiabatic compression

p_a is Amplitude of compression swing

γ is Ratio of specific heats

D_h is Hydraulic diameter $D_h = 4V_0 / A_0$

ω is angular velocity

α_0 is Mean thermal diffusivity

Pe_h is Peclet number (based on D_h)

L_s is Stroke

W_{adiab} is given by

Pe_h is defined by

$$W_{adiab} = \frac{p_0 \cdot V_0}{\gamma - 1} \left(\frac{1 + r_v}{2} \right)^{\gamma - 1} (1 - r_v^{(1 - \gamma)})$$

$$Pe_h = \frac{V_p \cdot D_h}{\alpha_0}$$

r_v is Volume ratio

V_p is Mean piston speed

Notes:

1. In the 1987 paper the authors recommend that for the best fit of their data the cylinder diameter D should be used for D_h – i.e. a hydraulic diameter based only on the cylinder wall area. Given that the fit is not greatly improved it appears safer to stick with the total area definition of D_h as this appears to have a stronger physical basis. It is likely that using the cylinder diameter alone will give worse results where the cylinder diameter is significantly greater than length of the spring volume.
2. In this paper the Peclet number Pe_h is defined in terms of the mean piston speed.

5.1.2 GAS SPRING LOSSES – KORNHAUSER 93

In the paper of 1993 Kornhauser no longer uses the non-dimensional loss constant L_{nd} but expresses the loss directly using the same equation from the 87 paper:

$$Loss \ (\dot{w}) = \frac{\pi}{2} \cdot p_0 V_0 \cdot \left(\frac{p_a}{p_0} \right)^2 \cdot \frac{(\gamma - 1)}{\gamma} \cdot \frac{1}{y} \left(\frac{\cosh(y) \cdot \sinh(y) - \sin(y) \cdot \cos(y)}{\cosh^2(y) - \sin^2(y)} \right)$$

All the parameters are defined as above except that the Peclet number is defined differently. The new Peclet number is defined by:

$$Pe_{\omega} = \frac{\omega D_h^2}{\alpha_0}$$

Kornhauser here returns to using the conventional definition of hydraulic diameter:

$$D_h = 4V_0 / A_0$$

The parameter y is now related to the Peclet number by:

$$y = \left(\frac{Pe_{\omega}}{8} \right)^{1/2}$$

It will be seen that this does not affect the definition of y in terms of D_h , ω and α_0

The reason for adopting a different definition of Peclet number was that it was found it correlated a wider a range of data including those for finned geometries.

The improved correlation suggested is achieved by replacing y in the equation given above with a modified parameter y' defined by:

$$y' = 1.20y^{0.86} = 0.49Pe_{\omega}^{0.43}$$

6 DESCRIPTION OF EXPERIMENTS

6.1 Experimental Rig

For the measurements of compression losses in a simple gas spring geometry the experimental setup shown in figures 1 and 2 was retained. Figure 1 is a photograph of the test rig. Figure 2 is a part section showing the shape of the compression space, the installation of pressure transducers and the piston cylinder assembly.

The test rig consisted of a piston/cylinder assembly driven by a linear motor. At maximum stroke the top of the piston reaches the end of the cylinder. Mounted on top of this assembly is a short cylindrical volume and a cylinder head which make up the gas spring fixed volume. The piston reciprocates within the cylinder causing the gas to undergo cyclic compression/expansion processes.

The position of the piston was measured using a commercial non-contacting LVDT, allowing the gas volume to be determined as a function of time. Provision was made for pressure transducers to be fitted flush with the sides of the cylinder and in the cylinder head as shown. Two pressure transducers were installed in order to check whether varying gas temperature would affect the accuracy of the transducers. One transducer was an Entran transducer that was compensated to minimise thermal effects, the other was a Druck transducer which did not have equivalent compensation. It was found that the two transducers gave very similar results and the conclusion drawn was that for the range of measurements made, the varying gas temperature didn't affect the performance of the transducers.

The piston/cylinder assembly does not have a sliding seal and the leakage past the piston is controlled by using a small clearance. This use of a “clearance seal” has been generally adopted with linear compressors to avoid processes that will cause wear.

6.2 Determination of Compression Loss

The total power input into the gas volume is made up of two components. The major component is the compression loss in the gas that we are interested in. In addition to this there is also a seal loss due to leakage through the clearance seal:

$$P_{in} = P_{cl} + P_{sl}$$

Where P_{cl} is the compression loss and P_{sl} is the seal loss.

The seal loss cannot be measured independently and was calculated using measurements of the clearances seal flow properties together with pressure difference across it. The Reynolds numbers for the leakage flows that occur through clearance seals are generally very low and well into the laminar flow regime. If laminar flow is assumed then an expression can be derived that relates the seal loss to the pressure variation, gas properties and seal dimensions. For a sinusoidal pressure variation with constant seal geometry

$$P_{sl} = \frac{\pi.D.r^3.p_a^2}{24.\mu.L}$$

Where D is the diameter of piston, r is the seal radial clearance, μ is the gas viscosity, L is the seal length, p_a is the pressure amplitude of the gas volume (the pressure at back of seal is assumed to be constant)

In general the pressure is not exactly sinusoidal and the seal clearance may vary as the piston moves. In order to obtain more accurate values the variation of seal clearance with position was measured for steady flow conditions. The pressure variation at the back of the seal was also measured. The instantaneous values for power loss can be calculated and the mean loss found by integrating over a cycle. This is described more fully in Appendix 2.

The power input P_{in} into the gas due to the gas forces acting on the piston is given by

$$P_{in} = f.w = f.\oint PdV$$

Where P is the cylinder pressure, V is the cylinder volume and f is the operating frequency. P_{in} was calculated from measured values of the cyclic pressure and volume variations. These were obtained from instantaneous values of cylinder pressure and piston position after compensating for phase errors that are caused by the frequency response of the instrumentation. The method used for phase compensation was described in detail in the first report LOS-FR1 and is retained for the work described here.

The compression loss is then calculated from the values of P_{in} and P_{sl} obtained:

$$P_{cl} = P_{in} - P_{sl}$$

In order to investigate the spring loss over wide range of conditions it is necessary to vary the Peclet Number defined by:

$$Pe_{\omega} = \frac{\omega \cdot D_h^2}{\alpha_0}$$

It will be seen that for a fixed geometry D_h is also fixed and the Peclet number can only be varied by varying the frequency and the thermal diffusivity. Varying the stroke has very little effect on the Peclet number.

The thermal diffusivity varies from gas to gas and is generally inversely proportional to the pressure. A wide range of Peclet numbers can be covered by varying pressure, frequency and the type of gas. Table 1 gives the relevant properties for a number of gases

Table 1: The Thermal Properties of a Range of Gases

Gas	Thermal Conductivity W/mK	Gas Constant R J/kgK	Heat Capacity Cp kJ/kgK	Density at 1 bar, 300 K kg/m ³	Thermal Diffusivity m ² /s	Peclet No for 10 Hz, D _h =13 mm
N ₂	0.0259	296.74	1.041	1.123	2.21E-05	120
He	0.1560	2078.14	5.193	0.160	1.87E-04	14
Ar	0.0177	208.01	0.522	1.603	2.12E-05	125
Xe	0.0056	63.00	0.160	5.291	6.58E-06	404
Ne	0.0483	412.22	1.030	0.809	5.79E-05	46
H ₂	0.1858	4126.79	14.312	0.081	1.61E-04	17
CH ₄	0.0346	517.40	2.236	0.644	2.40E-05	111
Kr	0.0094	99.00	0.249	3.367	1.12E-05	237

Compression losses were measured for four gases: Helium, Nitrogen, Neon and Krypton. It will be seen that the variation in thermal diffusivity is useful for achieving a range of Peclet Numbers. Xenon would have also been used but was too expensive.

6.3 Experimental Results

6.3.1 Variation of Seal Clearance

The variation of seal clearance with piston position was determined by measuring the pressure drop and mass flow rate for steady flow through the seal. Several measurements were made for the range of piston movement.

The clearance r is calculated from the equation

$$\frac{dm}{dt} = \frac{\pi \cdot D \cdot \rho_m \cdot r^3}{12 \cdot \mu \cdot L} \cdot dp$$

The clearance was found to vary between 8 and 10 microns as is shown in Figure 3



Figure 3 Variation of Seal Radial Clearance with Piston Position

The variation was not found to be a linear function of position but for the purposes of applying a small correction to the seal loss calculation, assuming a linear function appeared to be better than making no correction at all

6.3.2 Measured Compression Losses

Compression Losses were measured for a range of gases pressures and frequencies. The values are compared with those predicted by both Lee's original equation and Kornhauser's modified equation, and y is given by:

$$y = aPe_{\omega}^b$$

For Lee's original equation: $a = \frac{1}{\sqrt{8}} = 0.354$ $b = 0.5$

For Kornhauser's modification $b = 0.49$ $b = 0.43$

6.3.3 Results for Helium

The gas spring losses measured for Helium are plotted in Figure 4. The results generally give good support to Kornhauser's modification of Lee's equation. The results show little scatter for the range of Peclet Numbers between 10 and 1000 and there is little evidence for any dependence on pressure. The scatter for the results for Peclet numbers between 1 and 10 is greater but measurement errors were becoming much more significant and the scatter is within expected range of error.

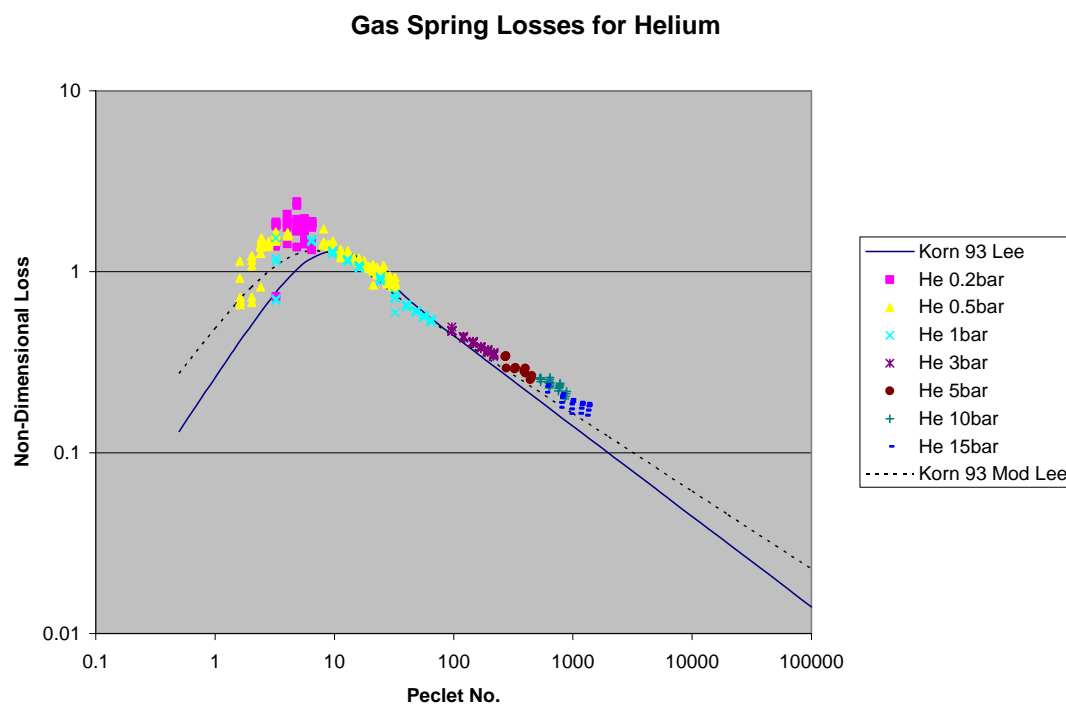


Figure 4: Gas Spring Losses for Helium

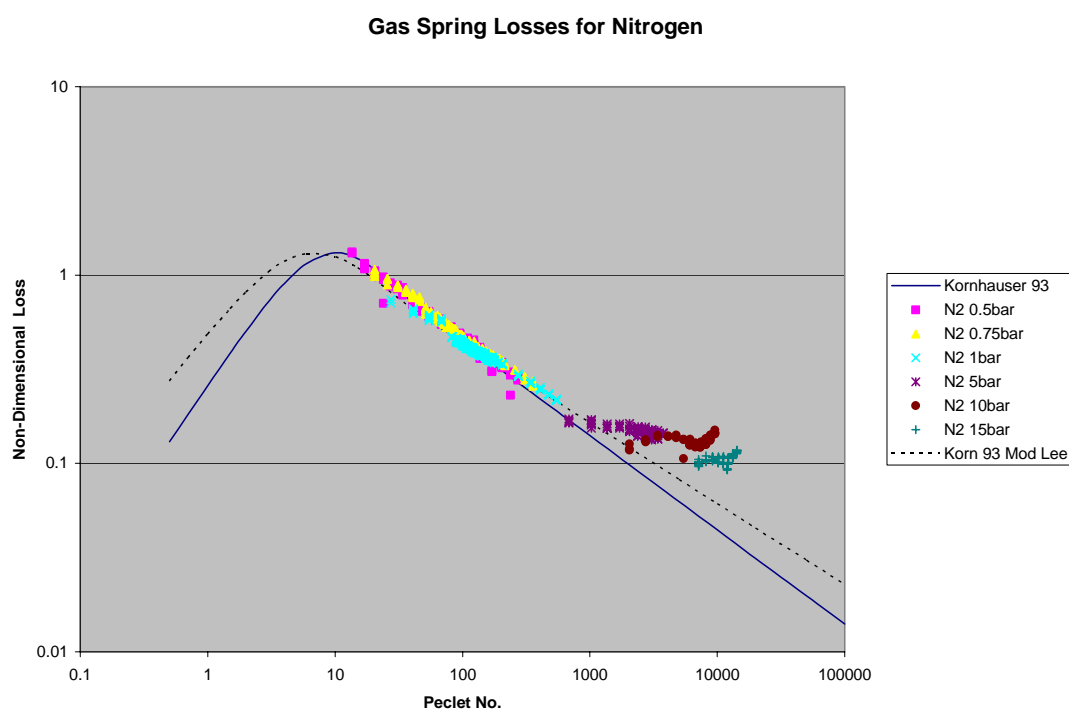


Figure 5: Gas Spring Losses for Nitrogen

6.3.4 Results for Nitrogen

The gas spring losses measured for Nitrogen are plotted in Figure 5. The results give good support to Kornhauser's modification of Lee's equation for the range of Peclet Numbers

between 10 and 1000. Again for this range the scatter is small and there is little evidence for any dependence on pressure. For Peclet numbers greater than 1000 there appears to be some deviation – the values are significantly higher than predicted by either Lee’s original equation or Kornhauser’s modification of it.

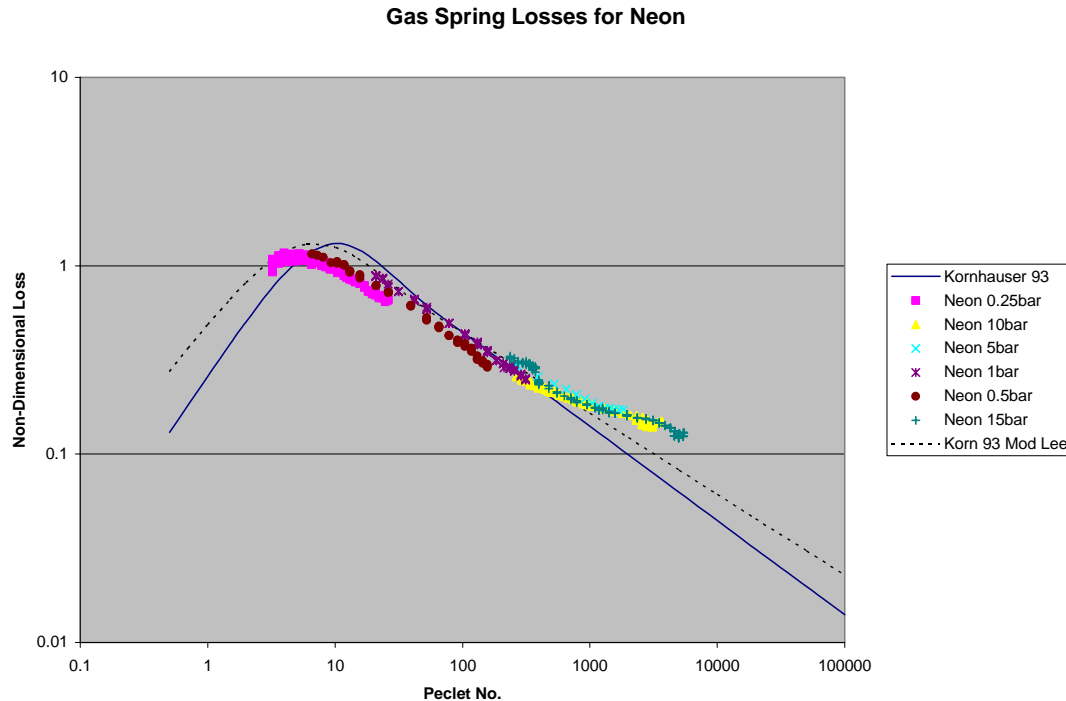


Figure 6: Gas Spring Losses for Neon

6.3.5 Results for Neon

The gas spring losses measured for Neon are plotted in Figure 6. The results are still closer to Kornhauser’s modified equation than Lee’s original but there is more deviation than for the Helium and Nitrogen results. It appears that there is a small dependence on pressure – the loss is relatively larger for the higher pressures. Again for Peclet numbers greater than 1000 there appears to be a significant deviation – the values are higher than predicted by either Lee’s original equation or Kornhauser’s modification.

6.3.6 Results for Krypton

The gas spring losses measured for Krypton are plotted in Figure 7. The results are again closer to Kornhauser’s modified equation than Lee’s original and there is more deviation than for the Helium and Nitrogen results. Again it appears that there is a small dependence on pressure – the loss is relatively larger for the higher pressures. For Peclet numbers greater than 1000 the deviation seen in the Nitrogen and Neon results appears to be repeated.

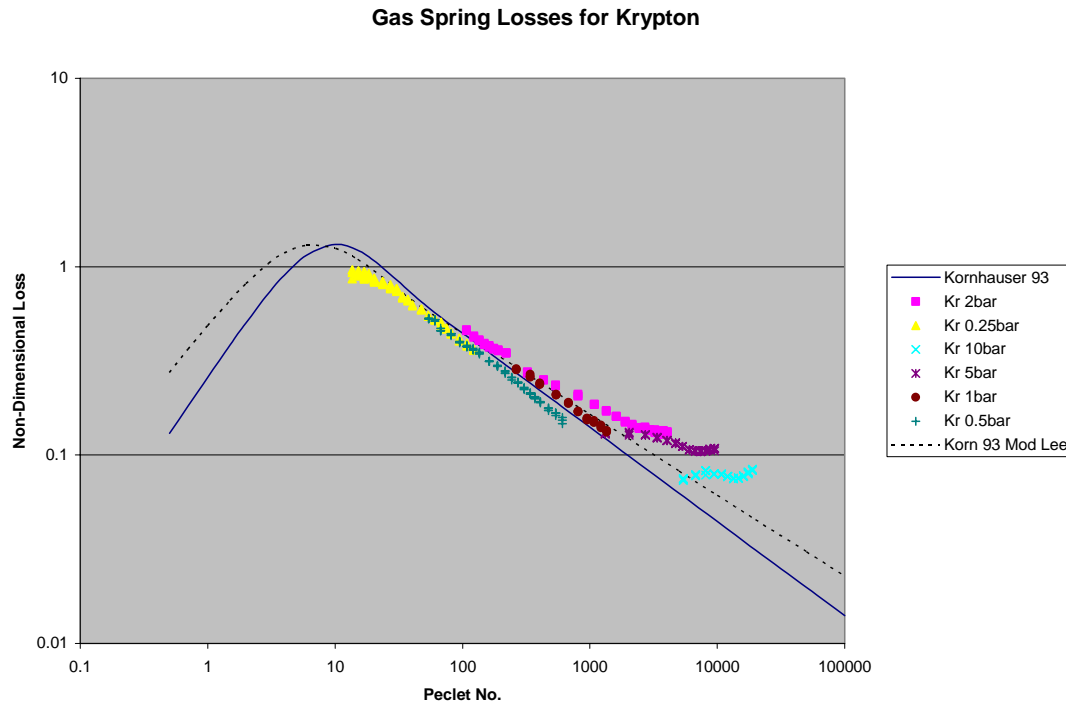


Figure 7: Gas Spring Losses for Krypton

6.3.7 Combined Results for All Gases

The results for all the gases are shown in Figure 8. It will be seen that altogether the results give good support to Kornhauser's modified equation. The spread of the results is really quite small given the range of pressures, frequencies and gases used.

The lowest Peclet number results (<10) were only obtained for Helium and Neon. Even allowing for the scatter on the helium results there does appear to be a difference in the maximum values of the non-dimensional loss although the Peclet numbers do correspond. There is no obvious explanation for this and because it is at the extremity of the data range more results are really required to demonstrate that it is a real effect.

At the highest Peclet numbers (>1000) all the gases (except helium for which this is not any data) show higher loss values than predicted by either of the analytic expressions.

At this point it is worth comparing the results with the correlation in Kornhauser's 93 paper. The baseline results for helium are shown in Figure 9. They show more spread for Peclet numbers between 10 and 100 but for high Peclet numbers there is little deviation.

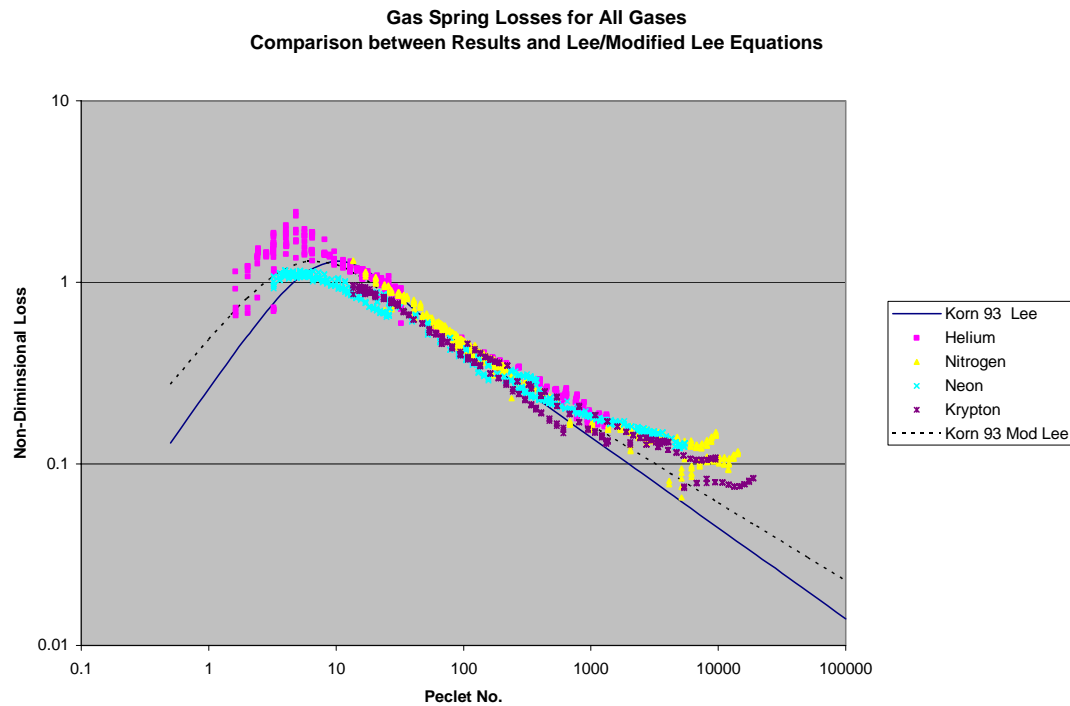


Figure 8: Gas Spring Losses for All Gases

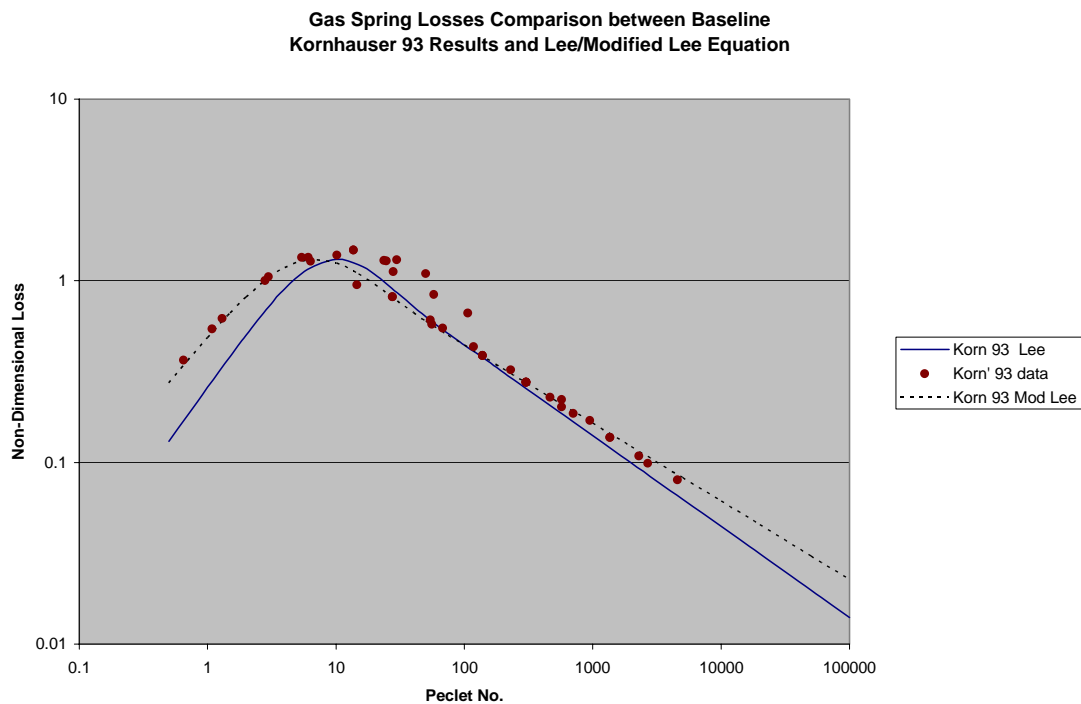


Figure 9: Kornhauser 93 Baseline Data

6.4 Discussion of Results

Overall the results given here give good support to those of Kornhauser. In particular the results for the range of Peclet number 10 to 1000 lie very close to the modified Lee expression. The low Peclet number results are somewhat inconclusive – there appears to be some deviation but the data is not good enough to be absolutely sure.

For the high Peclet number there appears to be an increase in loss over that predicted by the Lee model for all the gases except Helium. Kornhauser's baseline results, which are for Helium, do not show this tendency. In Kornhauser's 1993 paper there are sets of data that include those for other gases. These do demonstrate a similar tendency for the losses to be higher than those predicted by Lee's model.

The main property of helium that differentiates it from the gases measured is its low molecular weight and hence low density. This would suggest that the cause of the deviation at high Peclet numbers may be to do with a change in regime where the inertial properties of the gas become significant.

Assumptions Made in Lee's Model

The main assumptions made in Lee's model are:

- One dimensional planar model
- Small pressure variations – no density variation
- No convection/turbulence – heat transport is assumed to be entirely by conduction
- Appendix (small annulus around piston) effects ignored

Given the simple assumptions made and the degree to which the measurements depart from them, Lee's model is surprisingly successful at predicting gas spring losses. For example the ratio of pressure amplitude to mean pressure was as high as 0.3 without showing any obvious effect.

The effects of heat exchange and flow within the annulus around the piston (appendix effects) are ignored in Lee's model. If the appendix processes were significant then it might be expected that the results obtained with clearance seals, where leakage flows are significant, might reveal some effect. This does not appear to be the case for the results given here and suggests that Lee's model is still applicable.

The assumption that looks the weakest is the assumption that there is no turbulence or convective element to the heat transfer. Within the compression space there is gas flow which is at a maximum adjacent to the moving piston. Although the flow is very non-uniform and distinct compared with the normal flow describe in pipes etc, the basic phenomenon of a balance between inertial and viscous forces must still apply. It would therefore be expected that at some stage defined by an appropriate "Reynolds number" a transition would be reached in which a turbulent layer would be formed. This would immediately affect the heat transfer and consequent loss.

The Reynolds number for steady flow in a tube is defined by:

$$N_R = \frac{v.D.\rho}{\mu}$$

Where v is velocity, D is hydraulic diameter, ρ is density and μ is viscosity.

To define an equivalent Reynolds number for a gas spring we need only decide on how to define a velocity. The obvious candidate is the piston velocity or some function of it. For example the velocity could be defined as the mean velocity of a gas particle situated on the axis and halfway between the piston and the cylinder head. This in practice would be expected to be roughly half the mean velocity of the piston and hence is readily determined. There appears to be the possibility of correlating the deviation of the gas spring losses with such a Reynolds number.

6.5 Gas Spring Losses - Conclusions

The conclusions that can be drawn from these results are:

- For Peclet numbers in the range 10 to 1000 Kornhauser modified version of Lee's equation gives good results and can be used in applications involving typical clearance seals
- There appears to be a significant deviation from Lee's expression for all but the lightest gases (i.e. Helium and Hydrogen) where the Peclet number is above 1000. It would be of interest and value to investigate this further to discover the cause and whether there is a change in flow regime that can be defined by a type of Reynolds number.

7 RAPID RESPONSE TEMPERATURE MEASUREMENTS

7.1 Introduction

In the investigation of the heat transfer processes occurring in a component which is subject to rapidly varying pressures, there are two areas where, in principle, rapid response thermometry could be helpful:

- The measurement of boundary layer thickness would allow values to be determined for the enhanced heat transfer associated with turbulence and convection.
- The measurement of enthalpy flow into and out of a component would help understand the overall energy budget

The attainment of rapid frequency response using conventional sensors such as thermocouples and resistance thermometers requires the smallest sensors to be manufactured and handled. A lot of work has been done in this area (ref 3) but without going into any detail it is fair to say that improvements in response time are limited by the problems of handling very small

sensors and their lack of robustness. It is noted that as well as the response time there are two other significant issues involved with using conventional sensors.

- The effect the sensor and its mounting have on the measured temperature. The complete probe which contains the sensor can be intrusive and affect gas flows if care is not exercised.
- In some instances, e.g. for the measurement of enthalpy flow, a temperature distribution is required. Mapping out a distribution with a single probe has significant practical difficulties.

A novel approach to improving the response time without reducing sensor size has been successfully used in I.C. engines. This approach looks at the temperature variation of two sensors that have different thermal masses and hence response times. If they are assumed to be subject to the same temperature fluctuation it is, in principle, possible to use their combined outputs to compensate for the thermal masses and arrive at a true gas temperature.

In practice the presence of noise complicates the process and ultimately limits the frequency response that can be achieved. In the I.C. engine work a statistical technique utilizing Kalman filters was used to reconstruct the actual temperature variation. The details of this approach are too complicated to be presented here and are described in ref.3.

If the gas temperature variation in a compression loss component needs need to be measured then this dual sensor approach is attractive as it would allow higher frequency resolution for a particular sensor size. Given the deviation that the high Peclet numbers demonstrated in the gas spring loss measurements describe above it was considered worth trying to use this measurement technique to see if the deviation in measured results coincided with a deviation in the measured boundary layer thickness. If they did coincide it would add support to the idea that the increased loss was due to enhanced heat transfer.

7.2 Construction of Dual Thermocouple Thermometer

A dual thermocouple probe was made with the intention of fitting it in one of the spare pressure transducer ports in the simple gas spring rig described in section 6.1. The design of the probe is shown in figure 10.

Two proprietary freestanding thermocouples (~ 25 microns diameter) were installed in ceramic insulator housing. The ceramic housing was a proprietary component intended for the installation of fine wire sensors. It consists of a cylinder of ceramic material with six equally spaced holes running through its length. The thermocouples used four of the holes and were bonded into the housing with a “Loctite” anaerobic adhesive. Adhesive was used to fill the remaining two holes. The ceramic housing was bonded into an outer stainless steel body which incorporated the sealing and thread features of the pressure transducer the probe was intended to replace. The details showing how the ends of the thermocouples were terminated and connected to normal sized compensating cable are not included in Figure 10. As it turned out this task proved to be as difficult and time consuming as making the rest of the probe. An improvised arrangement was arrived at in which the connecting cables were bonded with araldite in such a position that a final soldering connection could be made with the minimum of manipulation.

After a complete probe was assembled, additional low viscosity anaerobic adhesive was wicked into joint gaps to increase strength and minimize any possible “appendix volumes”. The probe was successfully tested both for vacuum and pressure.

7.3 Operation of Dual Thermocouple Thermometer

Unfortunately when it came to operating the probe in the gas spring loss rig it was discovered that the probe could not be used as intended. For the dual thermocouple technique to work, it is necessary to have two junctions that have significantly different thermal masses – the differing thermal masses give information that allows the response times to be estimated. In the construction of the probe two identically sized thermocouples had been inadvertently used giving corresponding closely matched thermal masses. Remaking a new probe was considered but it was concluded that there was not sufficient time to do this and continue with the regenerator compressor loss test rig which was considered more important.

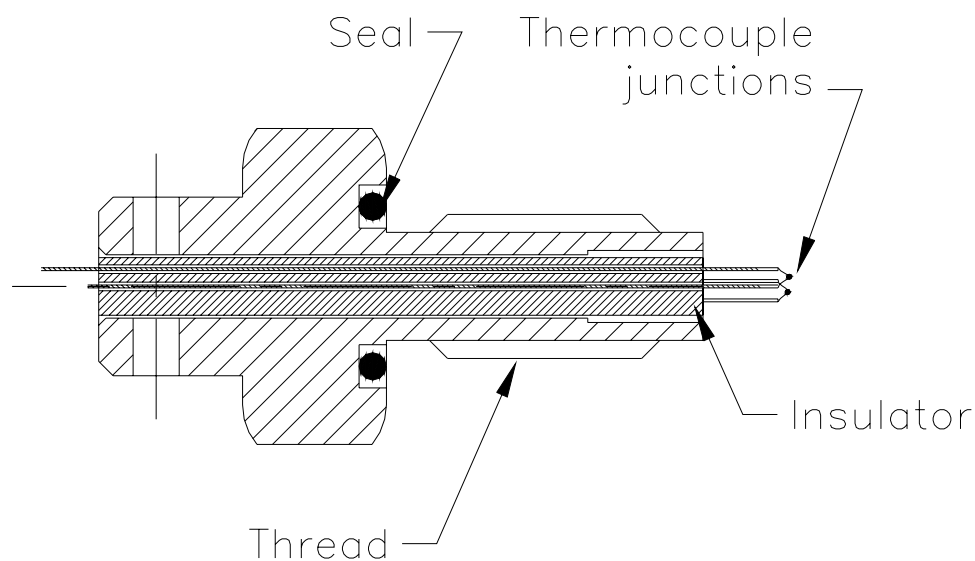


Figure 10: Design of Dual Thermocouple Probe

7.4 Dual Thermocouple Thermometer – Conclusions

Although the work described here did not get as far as successfully using this approach, it remains an attractive option – there appears to be no reason for not to work in this application. However, the implementation of this technique is quite involved, particularly if it is desired to measure the temperature distribution. The decision to use this technique is not a light one – the values of the results need to be significant to justify it.

8 USE OF COMPRESSION LOSS MEASUREMENTS TO DETERMINE HEAT TRANSFER COEFFICIENTS

8.1 Introduction

When this program to look at compression losses was originally considered, the intention was to investigate how the compression losses changed as components were added to a simple gas spring geometry. It was believed that a comparison could be made that would allow any change to be attributed directly to the loss characteristics of the added component.

With more detailed consideration this approach is clearly naïve and flawed. To demonstrate this, it is useful to produce a simple visual model of the processes that occur. This helps to identify the more obvious problems and to suggest possible new approaches.

8.2 Simple Visual Model

The simple gas spring described in section 6 has a loss that is principally determined by the Peclet number. This value characterizes the heat transfer conditions in the gas volume i.e. whether significant heat can be transported in the period of the compression/expansion processes.

For very small Peclet numbers (<0.1) the heat transfer rate throughout the compression volume is high enough to result in isothermal compression and expansion processes. The heat transfer that occurs between the wall and the gas can be assumed to involve all the gas in the compression volume. At the other extreme, for very high Peclet numbers (>1000) the heat transfer rate within the gas is too small to allow any significant heat transport across the gas. The result is that only a thin layer of gas close to wall can exchange heat with it. The bulk of the gas forms a core that undergoes isentropic compression and expansion processes. For intermediate values the picture is not so simple and both the temperature gradients and rates of heat transfer are significant. As this range ($Pe \sim 10$) is where the maximum loss occurs it is usually avoided.

At the extremes, a Peclet number tending to infinity approaches a reversible adiabatic gas spring whilst a Peclet number tending to zero approaches a reversible isothermal gas spring.

The regime that is of most interest in typical Stirling cycle coolers is the Peclet number range where $Pe \sim 100$ or higher. For this range the compression volume can be visualised as consisting of two components undergoing a compression process:

- A loss-free inner core where the gas undergoes isentropic compression and expansion.
- A boundary layer where the gas undergoes irreversible polytropic processes that convert net work from the piston to heat that is rejected to the wall.

This simple model is illustrated in Figure 11.

The Peclet number characterizes the heat transfer that occurs within a period of the compression cycle. In addition to this there is also the net heat transfer that occurs over many cycles - the dc component. It will be seen that the core gas has no net input of work or heat hence equilibrium is only achieved when there is no net heat flow into or out of the core. For equilibrium the mean gas temperature will be reasonably close to the wall temperature but it will not be the same and it will take a number of cycles to be reached.

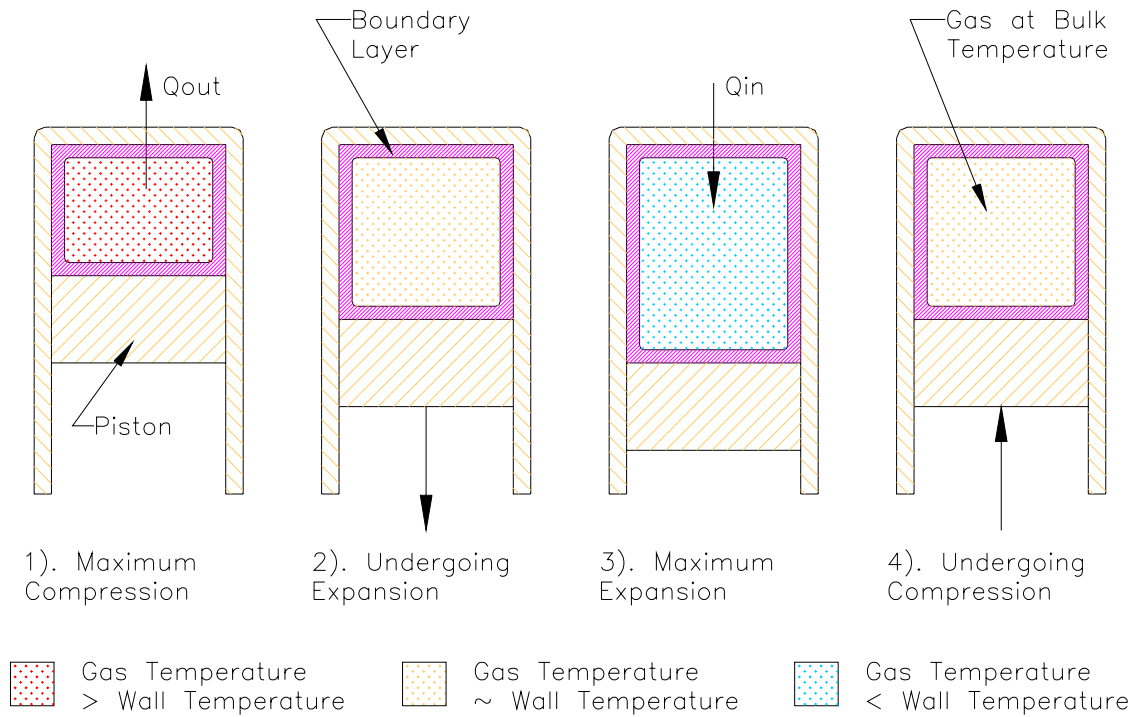


Figure 11 Simple Visual Model for High Peclet Number Gas Spring

8.3 Work and Entropy for Simple Gas Spring

Once equilibrium has been established the gas temperatures settle into a periodic variation where the temperature at a particular point is the same from one cycle to the next. Over a cycle there is no change in internal energy. The net heat flow out must therefore equal the work done on the spring i.e. the spring loss:

$$\oint dU = 0 \quad \text{Hence} \quad Q_{out} = \oint dQ = \oint dW = W_{in}$$

The mean temperature and temperature variation of the gas is determined by the requirement that the heat transfer results in the appropriate net heat loss:

$$Q_{out} = \oint h(t) \cdot (T(t) - T_w) \cdot dt$$

$h(t)$ is an effective heat transfer coefficient, $T(t)$ is an effective gas temperature for the boundary layer. T_w is the wall temperature.

The net change of entropy for the system is given by considering the heat loss from the wall assumed to occur at T_w

$$dQ = T \cdot dS \quad \text{Hence} \quad dS = \frac{Q_{out}}{T_w}$$

It is useful to also consider the changes in entropy of the gas. The change in entropy will be given by:

$$dS = \oint \frac{dQ(t)}{T(t)} dt$$

where $dQ(t)$ is the heat or leaving entering the gas boundary layer which is assumed to be at an effective temperature $T(t)$. This can be evaluated by splitting the integral into the two phases where heat is either entering or leaving the gas.

For the period where heat is leaving the gas the entropy of the gas decreases and we can define an effective gas temperature T_1 by

$$dS_1 = \int \frac{dQ}{T} = -\frac{Q_1}{T_1} \quad \text{where heat } Q_1 \text{ is the total heat leaving the gas}$$

Likewise the entropy increases whilst heat is entering the gas and we can also define an effective temperature for this process

$$dS_2 = \int \frac{dQ}{T} = \frac{Q_2}{T_2} \quad \text{where heat } Q_2 \text{ is the total heat entering the gas}$$

Two new temperatures can be defined as

$$T_3 = \frac{T_1 + T_2}{2} \quad \Delta T = \frac{T_1 - T_2}{2}$$

T_1 and T_2 can now be expressed as $T_1 = T_3 + \Delta T$ and $T_2 = T_3 - \Delta T$

For a closed cycle where the gas returns to initial state $\oint dS = 0$

Hence $dS_1 + dS_2 = -\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0$ or $Q_1 \cdot T_2 = Q_2 \cdot T_1$

Substituting expressions for T_1 and T_2 gives:

$$Q_1 \cdot (T_3 - \Delta T) = Q_2 \cdot (T_3 + \Delta T) \quad \text{or} \quad T_3 \cdot (Q_1 - Q_2) = \Delta T \cdot (Q_1 + Q_2)$$

For values of ΔT that are small compared with the absolute values of T_1 and T_2 , we can equate $T_3 \cdot (Q_1 - Q_2)$ with $T_w \cdot Q_{out}$. Hence the power dissipated W is given by

$$W = Q_{out} = \frac{\Delta T}{T_w} \cdot (Q_1 + Q_2)$$

This shows that the power loss is proportional to both the total heat transferred and the effective temperature difference. Referring to the Peclet number it can be seen that for a low Peclet number the heat transferred is high but the temperature difference is low resulting in a

low loss. The converse of this is that at high Peclet numbers the temperature difference is high but the heat transferred is low – again resulting in a small loss.

The result for the power loss given above can also be obtained by looking at the loss of exergy in terms of entropy creation. The Gouy-Stodola (ref. ⁷) theorem states that the loss of exergy and hence work dissipated is related to the entropy generation ΔS by the relationship:

$$W = \Delta S.T_w$$

The entropy generation for a transfer of heat Q through a temperature difference of ΔT at a temperature of T_w is given by

$$\Delta S = Q \cdot \frac{\Delta T}{T_w^2} \quad \text{Hence } W = T_w \cdot (Q_1 + Q_2) \cdot \frac{\Delta T}{T_w^2} = \frac{\Delta T}{T_w} \cdot (Q_1 + Q_2)$$

8.4 The Addition of Extra Components to a Simple Gas Spring

When components are added to the basic gas spring the system immediately becomes more complicated. Even if it is assumed that the two component model for high Peclet numbers can be retained, it is clear that the core gas is no longer homogenous and in general it will not be loss free. For example Figure 12 shows an arrangement where the plain cylinder head of the gas spring is replaced with a volume with good heat exchange maintained at a fixed temperature T_{hx} . It can be assumed that once gas enters the heat exchange volume it rapidly attains the temperature T_{hx} and whilst it stays in the heat exchanger any compression or expansion process will be isothermal. Gas leaving the heat exchanger will leave with a temperature of T_{hx} and thereafter its temperature will vary according to the expansion and the degree of mixing.

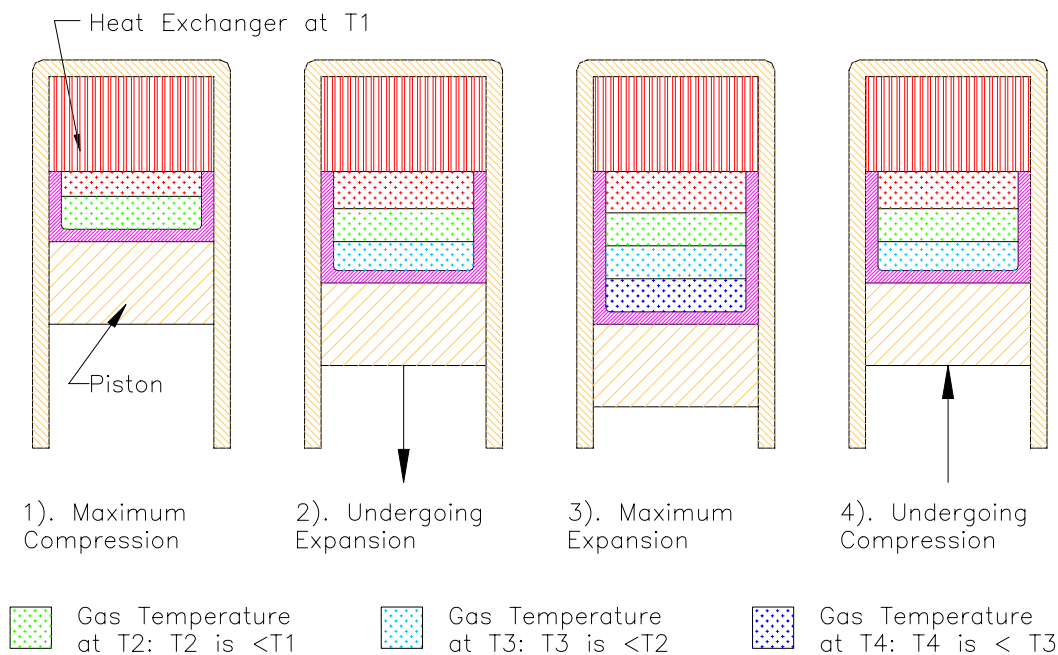


Figure 12. Simple Visual Model of Gas Spring With Added Heat Exchange Volume and Stratified Flow

The gas spring loss is now not just a function of the Peclet number. Two other parameters need to be defined to give a complete description even in this very simple model:

- The ratio of the heat exchanger temperature to the wall temperature.
- The degree of mixing that occurs in the gas as it leaves the heat exchanger

8.5 No Mixing – Stratified Flow

In figure 12 the gas is assumed to remain stratified. Starting at the point of minimum volume, it is seen that as successive layers leave the heat exchanger they expand and cool. The layers that leave first expand and cool the most. Hence, without mixing, a temperature gradient is established with the coolest gas adjacent to the piston. As all the gas expands from the same initial temperature it is clear that the mean gas temperature throughout the expansion is significantly lower than the heat exchanger temperature. If no heat transfer or mixing occurs in the gas during the cycle then during the compression phase the gas layers will retrace their paths and on arriving at the heat exchanger they will already be back to the heat exchanger temperature. As no heat is transferred between the gas and the heat exchanger the process is, in principle, isentropic and hence reversible.

However, when compared with the simple gas spring there is a significant difference. For the simple gas spring the core gas could be assumed to have a uniform temperature and its mean temperature was determined only by the requirement that the net heat into and out of the gas must be zero. For the example shown in figure 12 the temperature is non-uniform. It is also clear that the mean temperature is determined by the heat exchanger temperature and can be very different to the wall temperature. The temperature gradient in the boundary layer is therefore likely to be different.

It would be possible to arrange for the mean gas temperature in the cylinder to be close to the wall temperature by setting the heat exchanger temperature to be appropriately higher than the wall temperature. However the variation of the gas temperature across the cylinder volume would still affect the gradient in the boundary layer.

8.6 Complete Mixing

If complete mixing is assumed to occur in the cylinder as the gas leaves the heat exchanger then the core gas temperature is uniform and can be described by a single temperature. In this case the compression and expansion processes cannot be reversible as on the compression stroke the gas particles will not retrace their steps and most will arrive at the heat exchanger with temperature differing from T_{hx} . There will be heat exchanged between the gas and the heat exchanger over a finite temperature difference. The overall cycle therefore cannot be isentropic and an additional loss mechanism has been introduced as a result of the irreversible mixing process.

8.7 Summary

It is concluded that the addition of any heat exchanging components will have a significant effect on the losses occurring in the compression cylinder. The original idea of measuring the change of compression loss and attributing this loss to added components appears to be flawed. There remains the possibility of establishing some means of accounting for any change but this does not look realistic. There are number of effects that which appear to be unavoidable and difficult to quantify:

- If any mixing occurs there is automatically a loss associated with this mixing
- Any mixing occurring would inevitably affect the boundary layer and alter the boundary layer loss.
- If a stratified flow is maintained then there will be a significant temperature gradient in the core gas that will affect the boundary layer losses.

9 DIFFERENTIAL MEASUREMENTS – A NEW APPROACH

9.1 Introduction

It is clear from the above that if the measurement of compression losses is to be used to determine heat transfer losses in various components then a means must be found to ensure that the processes occurring in the cylinder are varied as little as possible. An approach that suggests itself is a differential one where components are not added but that their extent is varied. This approach is best illustrated with an example.

Figure 13 shows an arrangement where a compression space, comprising a piston and cylinder, is connected in series with three other components that are respectively a first heat exchanger, a regenerator volume and a buffer volume. The regenerator volume is designed so that it can readily be filled with different combinations of regenerator mesh. In Figure 13 two combinations are shown. In the top diagram there are two equal lengths of regenerator mesh, one either side of a length x_1 of mesh 2. In the lower diagram the arrangement is the same but the length of regenerator mesh 2 is now x_2 – i.e. the proportion of each regenerator mesh has been significantly changed.

If the compression losses were measured for these two cases for the same operating conditions, is it possible to deduce anything about the regenerator compression losses? To answer this we need to look at the various other loss components and consider if they are likely to vary significantly.

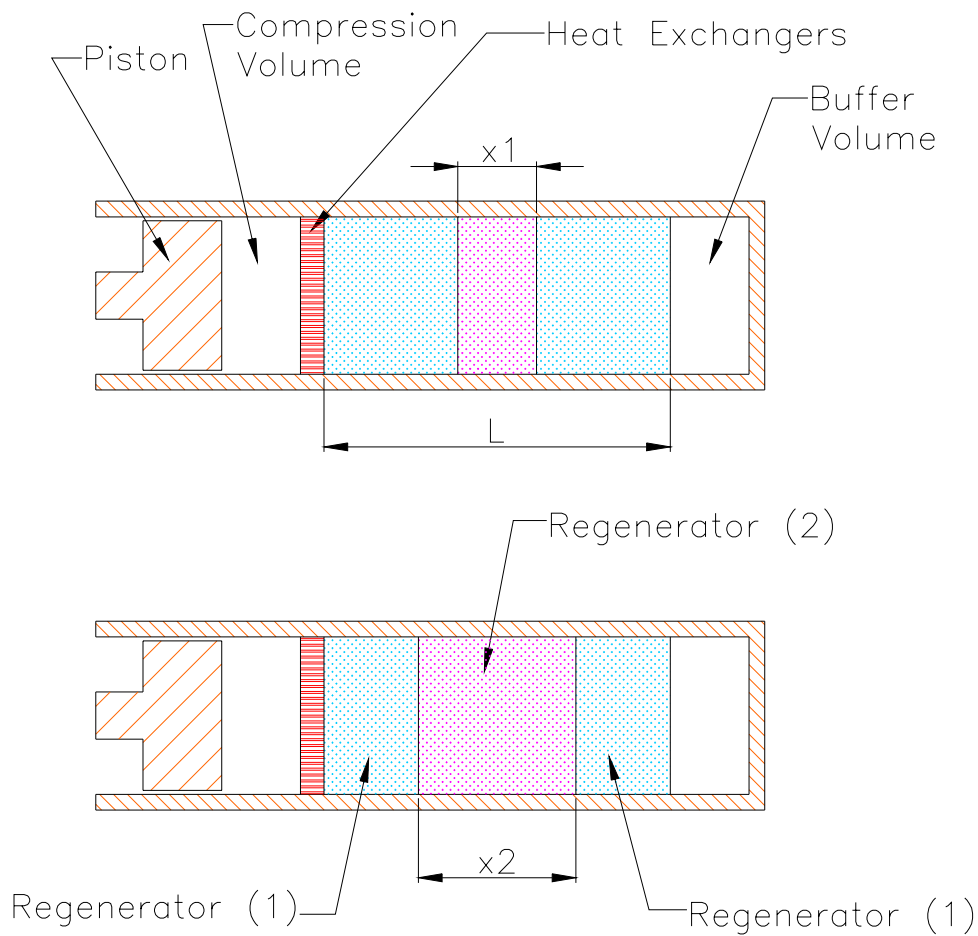


Figure 13. Differential Approach to Compression Loss Measurement

9.2 Variation of Various Components Of Compression Loss

Although there are a number of different components that can be identified, estimating their magnitude would be problematic, but deciding whether they are likely to change is fortunately a much easier task. A useful tool for doing this is the Gouy-Stodola theorem (ref. 7) already referred to above. This states that the loss of exergy and hence lost work associated with any entropy creating process is given by

$$W_l = \Delta S.T_w$$

Where: W_l is the loss component, ΔS is the entropy created in the component and T_w is the reject temperature.

For cases where the main source of entropy creation is the transfer of heat through a relatively small temperature difference at a temperature close to an ambient reject temperature T_0 then the loss can be expressed as:

$$W_l = \frac{\Delta T}{T_0} \cdot Q$$

Where: ΔT is the temperature difference and Q is the heat transferred through ΔT

Note: Q is not a net heat transferred, if the flow of heat is reversed then ΔT is also reversed so that the loss still has the same sign. This becomes clear when the integral form is considered:

$$W_l = \frac{1}{T_0} \oint \Delta T(t) \cdot \frac{dQ(t)}{dt} dt$$

If we now consider the arrangement shown in Figure 13 with a view to looking at the change in entropy generation that might occur in each component, then it is useful to trace the net flows of both entropy and enthalpy in the system.

Figure 14 shows how the net enthalpy flow is simply from the compressor piston to the heat exchanger. The net enthalpy flow between the heat exchanger and the regenerator and buffer volume is zero unless there is a heat input or out put in any of these components. As they will be close to the heat exchanger temperature or better still if they insulated any heat exchange across the boundary of the system will be very small.

Figure 14 also shows the entropy. Entropy is generated in all the components where irreversible processes are occurring. The entropy generated can only be rejected from the system where there is rejection of heat and this only occurs in the heat exchanger. There is therefore a migration of the entropy generated to the heat exchanger. At the heat exchanger there is additional entropy generated by the temperature difference involved in rejecting the heat to the heat sink at T_0 . A final quantity of entropy is rejected out of the system.

In Figure 14 components $\Delta S1$ to $\Delta S5$ represent the quantities of entropy generated in the various components excluding the heat exchanger and include both heat transfer and pressure drop contributions. The component ΔSh is the entropy generated in the heat exchanger. They are summed to give a total entropy ΔS_{Tot} .

The quantities $\Delta S1$ to $\Delta S5$ are generated by irreversible processes occurring within the components of the system due to the cyclic pressure variation. They can be independent of each as far as the processes are independent. The entropy generated in the heat exchanger has two components. One component is directly due to the cyclic pressure variation; the other however is due to entropy created in rejecting all the heat from the system through a finite temperature difference. The latter component is clearly dependent on the quantities of entropy created in all the other components.

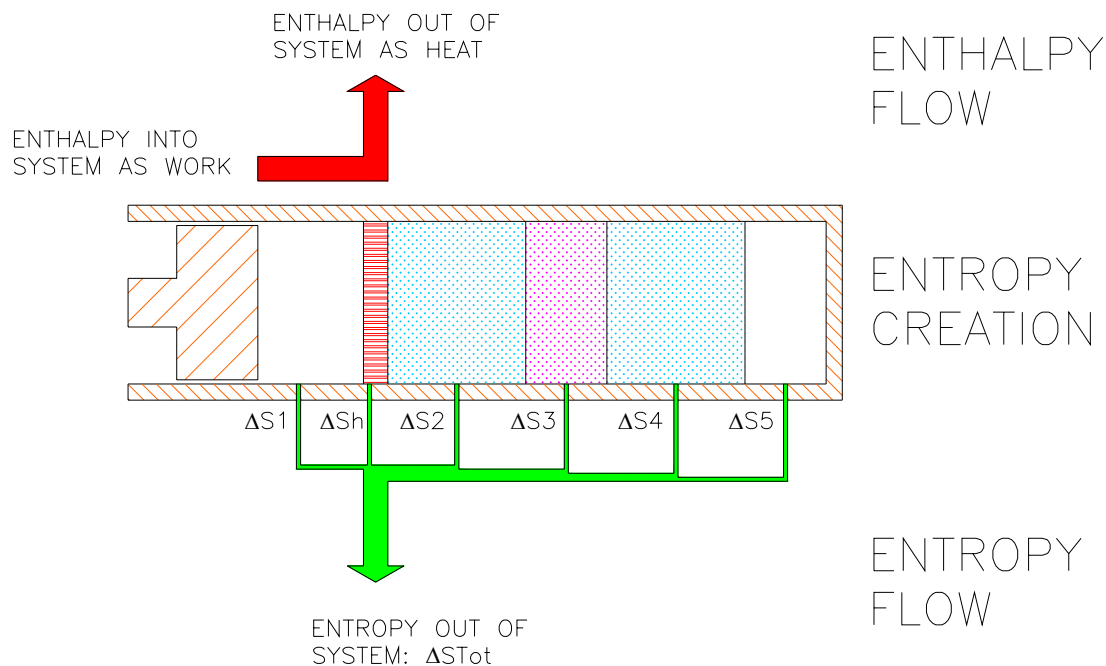


Figure 14. Enthalpy and Entropy Flow In System

To take this model further some initial simplifying assumptions will be made:

- The porosity of the two types of regenerator will be same. (This allows both the pressure variation and volume variation to be kept constant at the same pressure. This assumption can be met but it is limiting and methods for relaxing it need to be considered)
- The pressure variation is kept constant.
- The volume variation is kept constant
- The mean pressure is kept constant
- The frequency is kept constant
- The Peclet numbers, flow conditions and temperature gradients in the compression and buffer volumes will be assumed to remain the same. Hence the entropy generation will be kept constant.
- The entropy generated in the heat exchanger will be assumed to be only due to the rejection of the heat from the system. The generation of entropy due to cyclic pressure variation in this component will be small and can be ignored.
- The heat transfer coefficient in the heat exchanger remains constant i.e.

$$Q = h\Delta T$$
- The entropy generation in a regenerator component for a given pressure variation etc will be proportional to the volume of the regenerator.
- The temperature gradient along the regenerator is small hence any heat transfer entropy creation due to tidal flow can be neglected.

9.3 Analysis of Entropy and Enthalpy Flow in Model

ΔS_c is Entropy created that is assumed to remain constant for changes in Regenerator proportions. This includes entropy created in compression and buffer volumes. It can also include any end effects that occur in the regenerators where the temperature differences change.

ΔS_{R1} is the entropy created in regenerator 1 that is proportional to its length x :

$$\Delta S_{R1} = \varepsilon_1 \cdot x \text{ where } \varepsilon_1 = \frac{d(\Delta S_{R1})}{dx} \quad \varepsilon_1 \text{ is assumed to be constant}$$

ΔS_{R2} is the entropy created in regenerator 2 that is proportional to its length $(L - x)$:

$$\Delta S_{R2} = \varepsilon_2 \cdot (L - x)$$

$$\text{where } \varepsilon_2 = -\frac{d(\Delta S_{R2})}{dx}$$

ε_2 is assumed to be constant and L is the total length of both regenerators

$$\Delta S_R = \Delta S_{R1} + \Delta S_{R2} = \varepsilon_2 \cdot L + (\varepsilon_1 - \varepsilon_2) \cdot x$$

ΔS_{pd} is the entropy created by the pressure drop across the heat exchanger/regenerator assembly.

$$\Delta S_{pd} = \frac{W_{pd}}{T_0} \text{ where } W_{pd} \text{ is work lost in pressure drop and } T_0 \text{ is reject temperature.}$$

ΔS_{hex} is the entropy created in the heat exchanger as a result of rejecting the total heat Q out of the system to a heat sink at temperature T_0

$$\Delta S_{hex} = \frac{\Delta T \cdot Q}{T_0^2} \quad \Delta T = \frac{Q}{h_{hex}} \quad \text{Hence } \Delta S_{hex} = \frac{Q^2}{h_{hex} \cdot T_0^2}$$

$$\text{Total entropy generated is } \sum \Delta S = \Delta S_c + \Delta S_R + \Delta S_{pd} + \Delta S_{hex} = \Delta S_{Tot}$$

Using the Gouy-Stodola theorem and the fact that the compression loss power W_{cl} is equal to the heat rejected Q

$$W_{cl} = T_0 \cdot \Delta S_{Tot} = T_0 \cdot \Delta S_c + T_0 \cdot (\varepsilon_2 \cdot L + (\varepsilon_1 - \varepsilon_2) \cdot x) + W_{pd} + \frac{W_{cl}^2}{h_{hex} \cdot T_0}$$

If this equation is differentiated with respect to x then

$$\frac{dW_{cl}}{dx} = T_0 \cdot (\varepsilon_1 - \varepsilon_2) + \frac{dW_{pd}}{dx} + 2 \frac{W_{cl}}{h_{hex} \cdot T_0} \cdot \frac{dW_{cl}}{dx}$$

Rearranging this gives:

$$(\varepsilon_1 - \varepsilon_2) = \frac{1}{T_0} \left[\left(1 - \frac{2W_{cl}}{h_{hex} \cdot T_0} \right) \cdot \frac{dW_{cl}}{dx} - \frac{dW_{pd}}{dx} \right]$$

Thus it is possible to directly relate changes in the regenerator compression loss, and hence regenerator heat transfer, to measurements of overall power loss. It is also noteworthy that these measurements also allow the heat transfer coefficient for the heat exchanger to be determined. In the design and analysis of Stirling cycle machines such information is very valuable.

9.4 The Determination of Entropy Generation and Heat Exchanger Properties

The equation derived in the last section allows the heat transfer properties for both the heat exchanger and the regenerator to be investigated. The general procedure for a single data set is as follows:

- The compression loss and pressure drops are measured for varying values of x . The fill pressure, pressure variation, T_0 and frequency are kept the same. The volume variation might not be exactly the same because of some variation in regenerator porosity but small differences will not be very significant.
- The work lost in the pressure drops is determined from the pressure drop and volume flow rate calculated from the swept volume – some assumptions need to be made about the temperatures in the compression and buffer volumes but these do not need to be very accurate.
- The gradients $\frac{dW_{cl}}{dx}$ and $\frac{dW_{pd}}{dx}$ can be calculated and all the required quantities necessary to evaluate $\varepsilon_1 - \varepsilon_2$ are then available except h_{hex} . It is seen that $\varepsilon_1 - \varepsilon_2$ is expected to be independent of x hence the value of h_{hex} that results in a constant value can be found by trial and error.

This procedure will result in experimental values for h_{hex} and $\varepsilon_1 - \varepsilon_2$. It is noted that the last quantity does not give an absolute value of the entropy generation in the two types of mesh; instead it gives a comparative value. Although this may not be ideal it is nonetheless very useful.

The procedure described can be repeated for ranges of fill pressures, frequencies, volume/pressure variations etc including the use of different gases. It is also possible to repeat the measurements different heat exchangers and mesh combinations.

9.5 The Determination of Mesh Heat Transfer from Entropy Measurements

The values ε_1 and ε_2 were defined by

$$\varepsilon_1 = \frac{d(\Delta S_{R1})}{dx} \text{ and } \varepsilon_2 = -\frac{d(\Delta S_{R2})}{dx}$$

It is useful to relate these values to mean temperature differences occurring between the mesh and the gas. To this we will consider a very simple approximate cycle for gas undergoing cyclic compression and expansion processes:

- It is assumed that at the mesh volume is fixed and its temperature is constant at T_0 .
- At beginning of compression, gas pressure is $P1$ and temperature is T_0 .
- A temperature gradient ΔT is established and maintained for the compression to pressure $P2$. The pressure is maintained at $P2$ while the gas temperature returns to T_0

The enthalpy flow into the volume will be given by

$$\Delta H = c_p (T_0 + \Delta T). dm$$

Where c_p is constant pressure heat capacity and dm is increase in mass of gas.

The increase in internal energy of the gas will be given by:

$$\Delta E = c_v (T_0). dm \quad \text{here } c_v \text{ is constant volume heat capacity}$$

The heat transferred to the mesh is therefore given by:

$$Q = \Delta H - \Delta E$$

The increase in mass is calculated using the ideal gas equation and the initial and final state of the gas.

$$m_1 = \frac{M.P1.V}{R.T_0} \text{ and } m_2 = \frac{M.P2.V}{R.T_0} \quad \text{giving } dm = \frac{M.V(P2 - P1)}{R.T_0}$$

where M is molecular mass of gas and R is the Universal Gas constant

$$\text{Hence } Q = [c_p (T_0 + \Delta T) - c_v (T_0)] \frac{M.(P2 - P1).A.x}{R.T_0}$$

where A is effective cross sectional area.

For values of ΔT that are small compared with T_0 this can be simplified to:

$$Q = [(c_p - c_v).T_0] \frac{M.(P2 - P1).A.x}{R.T_0}$$

Q and ΔT for the expansion will be very similar so that the total entropy generated in volume over a cycle is given by:

$$\Delta S = \frac{2\Delta T.Q}{T_0^2} = \frac{2\Delta T}{T_0^2} [(c_p - c_v).T_0] \frac{M.(P_2 - P_1).A.x}{R.T_0}$$

$$\frac{d(\Delta S)}{dx} = \frac{2\Delta T}{T_0^2} [(c_p - c_v).T_0] \frac{M.(P_2 - P_1).A}{R.T_0}$$

Hence values of ε can be used to derive values for the mean temperature differences in the mesh.

9.6 Progress on Compression Loss Measurement

The components for a regenerator test rig were manufactured to an initial design that did not include a heat exchanger. Figure 15 shows how these components were to be installed on the test compressor. The analysis described above requires that a heat exchanger is installed to fix the temperature of gas entering the compression cylinder. The present rig is therefore not satisfactory for making compression loss measurements. However the modifications required to fit a suitable heat exchanger are small and it is intended to do this in a future program of work.

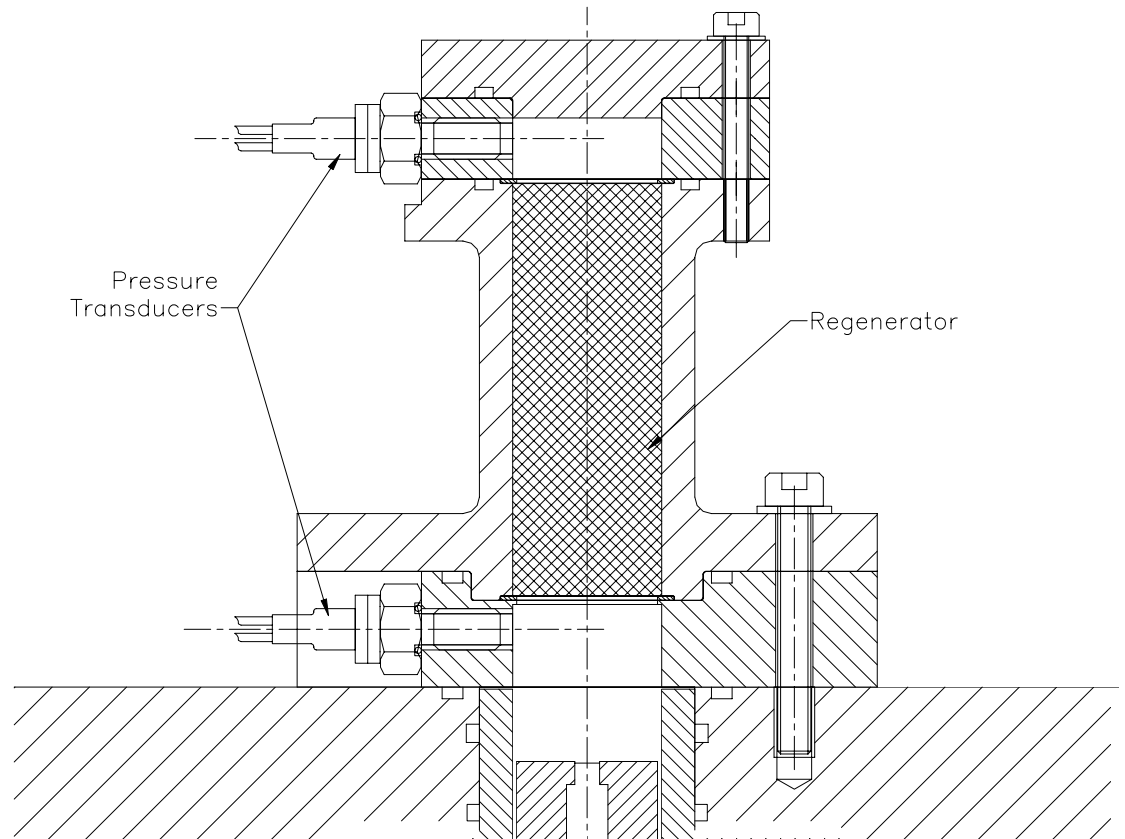


Figure 15. Initial Design Of Regenerator Test Rig

10 CONCLUSION

The aim of the work was to investigate various losses associated with compression and expansion processes typically found in cryocooler compressors. A detailed investigation of these losses was made using different gases over a range of frequencies, and the losses were estimated from the 'pdV' work done by the piston on the gas in the cylinder.

Having allowed for the pumping loss due to the clearance seal, the work done on the gas was evaluated, and this data was compared with expressions derived by Kornhauser and others using sliding seal compressors.

An important conclusion is that these expressions, derived for conventional sliding seal compressors, are valid for clearance seal machines, once the seal pumping loss has been allowed for. Other conclusions that can be drawn from these results are:

- For Peclet numbers in the range 10 to 1000, Kornhauser's modified version of Lee's equation gives good results and can be used in applications involving typical clearance seals.
- There appears to be a significant deviation from Lee's expression for all but the lightest gases (i.e. Helium and Hydrogen) where the Peclet number is above 1000. It would be of interest and value to investigate this further to discover the cause and whether there is a change in flow regime that can be defined by a type of Reynolds number.

The original idea of adding components to a simple gas spring in order to measure their compression losses was found to be flawed. However a more sophisticated approach, that involves varying the extent of a heat exchanger and relating this to changes in measured losses, does appear to be viable. This new approach requires a heat exchanger component that was not designed into the original test rig. It also requires the regenerator to be repacked several times for a set of results. It was not possible to obtain any measurements in this program but it is still considered worth pursuing in any future program

11 APPENDIX 1: HEAT TRANSFER LOSSES IN GAS SPRINGS: A COMPARISON OF EXPRESSIONS

A1.1. INTRODUCTION

There are five primarily relevant sources:

- **Kornhauser87** (Ref. 2) This is the ‘reference’ in the subject, and it compares the analytical solutions of the 1D heat transfer equations, obtained by Lee/Smith and Cooke-Yarborough/Ryden (CYR), to experimental data and other expressions. These 1D solutions seem to fit the data reasonably well.
- **Kornhauser93** (Ref 6). This extends the work reported in Kornhauser93 and gives an empirical modification to Lee’s expression.
- **CYR85** (Cooke-Yarborough and Ryden, ref 5).
- **Lee83** (Lee KP, Ref 4).
- **Cantelmi98** (Cantelmi, Gedeon, and Kornhauser, Ref 8) AA. This paper describes an analytical model including the effects of turbulence.

Unfortunately all the expressions use different notation. I will retain the original notation to avoid introducing errors. I will also illustrate the argument with a specific example having the following parameters:

Parameter	Lee	CYR	Kornhauser	Cantelmi	Value
Hydraulic diameter	-	h	D_h		0.01053 m
Hydraulic radius	r	-		r_H	0.005265 m
Mean pressure	P_0	P	P_0	P_0	1526192 Pa
Thermal diffusivity	α	-	α_0	α	1.2e-5 m ² /s
Frequency of operation	f	f	f	f	33.0 Hz
Ratio of specific heats	γ	-	γ	γ	1.66
Mean/wall temperature	T_0	T_0	T_0		300.0 K
Mean volume	V_0	V_a	V_0	v_0	6.884e-6 m ³
Pressure amplitude	P_1	Δp	P_a	P_1	3.6e5 Pa
Specific heat	c_p	c_p	c_p	c_p	5193 J/kg
Thermal conductivity	k	k	k	k	152e-3 W/mK

A1.2. EXPRESSIONS

A1.2.1. CYR

CYR base the power loss on a calculation of the bulk temperature of the gas, and then perform an entropy analysis on the resulting heat transfer. The general expression presented is:

$$\bar{E} = \frac{\omega(\Delta p)^2}{\rho_0 c_p T_0} \frac{1}{\alpha h} \left[\frac{\sinh(\alpha h) - \sin(\alpha h)}{\cosh(\alpha h) + \cos(\alpha h)} \right] \quad (1, \text{CYR B6})$$

Where: ω = angular frequency
 Δp = pressure amplitude (CYR eq. A4),
 ρ_0 = average density
 c_p = specific heat
 T_0 = average temperature
 $\alpha = \left(\frac{\omega \rho_0 c_p}{2k} \right)^{1/2}$ (NOTE: this is not the thermal diffusivity)
 k = thermal conductivity of the gas
 h = hydraulic diameter

A second expression is then presented:

$$\bar{E} = \frac{\omega(\Delta p)^2}{2\rho_0 c_p T_0} f(\alpha h) \quad (2, \text{CYR B6})$$

Where f is a function to be approximated in the adiabatic case ($\alpha h > 5$),

$$f(\alpha h) = \frac{1}{\alpha h} \quad (3)$$

and in the isothermal case ($\alpha h < 1$),

$$f(\alpha h) = \frac{(\alpha h)^2}{6} \quad (4)$$

There is an error in this paper since these expressions are not consistent. Consider the test case, each equation above gives the following values:

Eq. 1: Power loss, eq. (B6) 1st exp. : 1.56 W

$\alpha h = d/l$: 31

Eq. 3: adiabatic ($ah > 5$) approx : 0.78 W

Eq. 4: harmonic model: isothermal ($ah < 1$) approx : 3900 W

I.e. the approximated version is half the generalized expression. Taking a step back before the generalized expression there is an integral (CYR eq. B4/B5), which can be evaluated numerically. When evaluated numerically, the resulting power loss is:

$$\text{ave divT power : 0.78 W}$$

So it appears that the analytic expression is incorrect by a factor of 2.

A1.2.2. Lee

This paper again evaluates the bulk temperature of the 1D system but the power loss is calculated differently to CYR. In CYR it was an entropy calculation, here the volume corresponding to the temperature and pressure variations is calculated and this is used to evaluate the area of the PV loop (note that the temperature expression agrees with the CYR values). The expressions for the power loss are:

$$\Delta \dot{w} = -\frac{\omega}{2} p_1 V_0 \frac{\gamma-1}{\gamma} \frac{p_1}{p_0} \text{Im} \left[1 - \frac{\tanh z}{z} \right] \quad (5, \text{Lee } 12)$$

$$\Delta \dot{w} = -\frac{\omega}{4} p_1 V_0 \frac{\gamma-1}{\gamma} \frac{p_1}{p_0} \frac{\cosh y \sinh y - \sin y \cos y}{y (\cosh^2 y \cos^2 y + \sinh^2 y \sin^2 y)} \quad (6, \text{Lee } 12)$$

$$\begin{aligned} \text{Where: } z &= (1+i)y \\ y &= r/\lambda \\ \lambda &= \left(\frac{2\alpha}{\omega} \right)^{1/2} \end{aligned}$$

For the test case both expressions 5 and 6 agree:

$$\text{Eq. 5: 0.78 W}$$

$$\text{Eq. 6: 0.78 W}$$

This agrees with the CYR expressions once the error is taken into account.

A1.2.3. Kornhauser

This paper references both CYR and Lee, it states that the “result in both cases was”, in non-dimensional terms:

$$L_{nd} = \frac{P_0 V_0}{W_{adiab}} \frac{\pi}{2} \left(\frac{P_0}{P_0} \right)^2 \frac{\gamma-1}{\gamma} \frac{1}{y} \left[\frac{\cosh y \sinh y - \sin y \cos y}{\cosh^2 y - \sin^2 y} \right] \quad (7, \text{Kornhauser } 19)$$

The power loss is given by:

$$P_{loss} = fL_{nd}W_{adiab} \quad (8)$$

Where: $y = D_h \left(\frac{\omega}{32\alpha_0} \right)^{1/2}$

(There is also a definition in Peclet number, which will not be used here, but equates to the same value if the piston motion is a triangular waveform.)

There are two comments here:

1. Kornhauser states that CYR's, Lee's, and his expressions are all the same. This does not appear to be the case. Plotting the terms in cosh/sinh they are numerically identical above $y=4$ but below this value they appear to diverge. I have not attempted to analytically compare the expressions.
2. The definition of y in this case appears to be different to Lee's model. Lee's definitions give:

$$y = r/\lambda = \frac{d}{2\lambda} = \frac{d}{2\left(\frac{2\alpha}{\omega}\right)^{1/2}} = d\left(\frac{\omega}{8\alpha}\right)^{1/2}$$

i.e. A factor of 2 different.

With Kornhauser's definition of the y -factor the numerical power loss is:

Eq. 7: 1.56 W
 Adiabatic approx loss : 1.56 W
 Isothermal approx loss : 960 W

Interestingly whilst his theoretical results appear to be at odds with the papers he cites as his sources, they do agree better with his measured data.

A1.2.4. Cantelmi

This is the most recent paper and is based around extending the previous results (i.e. Lee's model), to situations with turbulent inflow (i.e. more like a real Stirling cooler). There are several interesting references, one of which is "Gedeon 1989" where it is stated that Gedeon "modified Lee's analysis in an attempt to predict the effects on inflow-produced turbulence." Unfortunately this is cited as a memo to R. Tew so isn't available, but apparently indicated that higher turbulence led to higher heat transfer, and a higher power loss. On the other hand Gedeon was a co-author of this paper so it can reasonably assumed that this paper is a direct continuation of this work. There is lots of additional material in this paper but a key result is an expression for the power loss:

$$\frac{w_{loss}}{P_0 v_0} = \frac{\pi}{2\beta} P_A^2 \frac{\gamma-1}{\gamma} (F_R - F_I) \quad (9, \text{Cantelmi 33})$$

Where:

$$\begin{aligned} \beta &= r_H / \delta_L \\ \delta_L &= \left(\frac{2\alpha}{\omega} \right)^{1/2} \\ P_A &= p_1 / p_0 \\ F_R, F_I &= \text{real and imaginary parts of a complex function, } F \end{aligned}$$

Now it is stated that when $F_R - F_I = 1$ the results "correspond to the laminar results obtained by Lee and Cooke-Yarborough and Ryden for $\beta \gg 1$ ". So we can easily calculate the value in the test case,

$$\begin{aligned} \text{Eq. 9: } 0.78 \text{ W} \\ \beta = 15.5 \end{aligned}$$

So this paper appears to agree with Lee and CYR but not the original Kornhauser paper.

A1.3. SUMMARY

There appears to be some question mark over the 1987 Kornhauser expression as it does not agree with either the two original sources or a more recent paper. Kornhauser's expression appeared to be incorrect because of the definition of the y -factor, although this expression has been used by him in at least two other papers/reports. It was also apparently validated by experiment. The more recent Cantelmi paper (also co-authored by Kornhauser), however, also appeared to be validated by experiment. In conclusion, there appears to be some further work to be done here!

The results in from our work agree better with Kornhauser's expression than with Lee's or CYR, both of which under-predict the loss.

12 APPENDIX 2: CALCULATION OF SEAL LOSS

SEAL POWER LOSS

The instantaneous mass flow rate through the seal is given by:

$$\frac{dm}{dt} = \frac{\pi.D.\rho_m.r^3(t)}{12.\mu.L}.dp(t) \quad \text{where } dp(t) = p_c(t) - p_0(t)$$

The radial clearance r is nominally constant but for a real compressor it is found to be a function of position and hence time

The volume flow at compression space end will be given by

$$\frac{dV}{dt} = \frac{\pi.D.\rho_m.r^3}{12.\mu.L} \cdot \frac{dp(t)}{\rho_c(t)}$$

The work done over a cycle given by:

$$W_{sl} = \oint p_c(t)dV = \oint p(t) \cdot \frac{dV}{dt} \cdot dt$$

Hence the power loss is given by

$$P_{sl} = f \cdot \oint p_c(t)dV = f \cdot \oint p(t) \cdot \frac{dV}{dt} \cdot dt$$

If the instantaneous values of pressure and position are known then the clearance can be calculated and the instantaneous power loss calculated. The mean power loss can then be determined by averaging this value over a complete cycle.

SEAL POWER LOSS FOR SINUSOIDAL PRESSURE VARIATION AND CONSTANT RADIAL CLEARANCE

If the pressure variation can be assumed to be sinusoidal and the radial clearance is constant then an analytic expression can be derived for the mean power loss:

$$P_{sl} = f \cdot \oint p_c(t)dV = f \cdot \oint p(t) \cdot \frac{dV}{dt} \cdot dt = \oint \frac{\pi.D.r^3}{12.\mu.L} \cdot p_a^2 \cdot \sin^2(\omega t) \cdot dt$$

Integration gives:

$$L_s = \frac{\pi.D.r^3 \cdot p_a^2}{24.\mu.L}$$

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